Initiated Wave Field

Initiated Wave Field (Pattern).

Free surface displacement generated by plate motion.

General description.

Free surface elevation in the open fluid $\mathcal{F}$ equals to a sum of the incident wave elevation and the additional wave elevation, generated by the plate motion

$$\zeta = \zeta^{inc} + \zeta^{pm}. \tag{1}$$

The total potential in $\mathcal{F}$ is also represented as a sum of the incident wave potential and the potential of waves, arising from the plate presence

$$\phi^{\mathcal{F}} = \phi^{inc} + \phi^{pm}, \tag{2}$$

here ϕ^{pm} is the classical diffraction potential plus radiation potential.

In polar coordinates

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} + \frac{K}{4\pi}\int\limits_{\mathcal{P}} \left\{\mathcal{D}\Delta^2 - \mu\right\} w(r,\theta)\,\mathcal{G}(\rho,\varphi,0;r,\theta,0)\,dS. \tag{3}$$

$dS = r\,dr\,d\theta$

Circle

$\mathcal{F}$: $\rho > r_0$.

IWD

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - \pi i r_0 \sum_{m=1}^{M} \frac{k_0^2}{(k_0^2-\kappa_m^2)}\left(\mathcal{D}\kappa_m^4 - \mu\right)$$
$$\times \sum_{n=0}^{N} a_{mn}\left[k_0 J_{n+1}(k_0 r_0) J_n(\kappa_m r_0) - \kappa_m J_n(k_0 r_0) J_{n+1}(\kappa_m r_0)\right] H_n^{(1)}(k_0\rho)dk. \tag{4}$$

FWD

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - \pi i K r_0 \sum_{m=1}^{M}\sum_{i=0}^{M-3} \frac{k_i^2}{(k_i^2-\kappa_m^2)(k_i^2 h - K^2 h + K)}\left(\mathcal{D}\kappa_m^4 - \mu\right)$$
$$\times \sum_{n=0}^{N} a_{mn}\left[k_i J_{n+1}(k_i r_0) J_n(\kappa_m r_0) - \kappa_m J_n(k_i r_0) J_{n+1}(\kappa_m r_0)\right] H_n^{(1)}(k_i\rho)dk. \tag{5}$$

Ring

We continue the analysis of plate-water interaction and consider the open water regions - main region $\mathcal{F}_0$ and the gap inside of the ring $\mathcal{F}_1$. The elevation $\zeta(\rho,\varphi)$ of the free surface can be computed by (1), where the value of ζ^{inc} may be obtained from the incident wave potential expression with the use of kinematic condition and the value of ζ^{pm} - from the following analysis of IDE in the water area.

If we are dealing with FWD case, we obtain from IDE the following expression for the free surface elevation

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - K\int_{-\infty}^{\infty}\sum_{m=1}^{M}\left(\mathcal{D}\kappa_m^4-\mu\right)\frac{J_n(k\rho)}{(k^2-\kappa_m^2)}$$
$$\times\sum_{i=0}^{M-3}\frac{k_i^2}{(k_i^2h-K^2h+K)(k-k_i)}\left[a_{mn}c_{mn}^{(1)}+b_{mn}c_{mn}^{(2)}\right]dk. \tag{6}$$

We notice that in the open water region $\mathcal{F}_0$ as $\rho > r_0 > r_1$ $J_n(k\rho)$ is splitted up into the half-sum of corresponding Hanlkel functons to close the contour of the integration, and to do so at the gap area $\mathcal{F}_1$ Bessel functions $J_t(kr_1)$, $t=n,n+1$, are splitted up as $\rho < r_1 < r_0$.

Finally, after the use of the residue lemma at the poles $k=k_i$, we obtain for the gap area:

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - \pi i\sum_{m=1}^{M}\left(\mathcal{D}\kappa_m^4-\mu\right)$$
$$\times\sum_{i=0}^{M-3}\frac{k_i^2K}{(k_i^2h-K^2h+K)}\frac{J_n(k_i\rho)}{(k_i^2-\kappa_m^2)}\left[a_{mn}f_{1,mni}^{(1)}+b_{mn}f_{1,mni}^{(2)}\right]dk, \tag{7}$$

where introduced functions $f_{1,mni}^{(q)}$ are

$$f_{1,mni}^{(q)} = r_0\left[kH_{n+1}^{(1)}(k_ir_0)H_n^{(q)}(\kappa_mr_0)-\kappa_mH_n^{(1)}(k_ir_0)H_{n+1}^{(q)}(\kappa_mr_0)\right]-$$
$$r_1\left[kH_{n+1}^{(1)}(k_ir_1)H_n^{(q)}(\kappa_mr_1)-\kappa_mH_n^{(1)}(k_ir_1)H_{n+1}^{(q)}(\kappa_mr_1)\right],\quad q=1,2. \tag{8}$$

If we are at the open fluid area $\mathcal{F}_0$ $\rho > r_0$. Then the contour of the integration is closed by splitting up of Bessel functions $J_n(k\rho)$ into the half-sums of corresponding Hankel functions. Finally, for the elevation in the open water area we obtained the following expression

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - \pi i\sum_{m=1}^{M}\left(\mathcal{D}\kappa_m^4-\mu\right)$$
$$\times\sum_{i=0}^{M-3}\frac{k_i^2K}{(k_i^2h-K^2h+K)}\frac{H_n^{(1)}(k_i\rho)}{(k_i^2-\kappa_m^2)}\left[a_{mn}f_{mni}^{(1)}+b_{mn}f_{mni}^{(2)}\right]dk, \tag{9}$$

where introduced functions $f_{mni}^{(q)}$ are

$$f_{mni}^{(q)} = r_0\left[kJ_{n+1}(k_ir_0)H_n^{(q)}(\kappa_mr_0)-\kappa_mJ_n(k_ir_0)H_{n+1}^{(q)}(\kappa_mr_0)\right]-$$
$$r_1\left[kJ_{n+1}(k_ir_1)H_n^{(q)}(\kappa_mr_1)-\kappa_mJ_n(k_ir_1)H_{n+1}^{(q)}(\kappa_mr_1)\right],\quad q=1,2. \tag{10}$$

In general, for infinitely deep water the procedure is the same, but with the pole $k=k_0$ of the water dispersion relation only. We derive the following equation for the free surface elevation in the open water region $\mathcal{F}_0$

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - \pi i \sum_{m=1}^{M} \left(\mathcal{D}\kappa_m^4 - \mu\right) \frac{k_0^2 H_n^{(1)}(k_0\rho)}{(k_0^2 - \kappa_m^2)} \left[a_{mn} f_{mn0}^{(1)} + b_{mn} f_{mn0}^{(2)}\right] dk \quad (11)$$

and in the gap region $\mathcal{F}_1$

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - \pi i \sum_{m=1}^{M} \left(\mathcal{D}\kappa_m^4 - \mu\right) \frac{k_0^2 J_n(k_0\rho)}{(k_0^2 - \kappa_m^2)} \left[a_{mn} f_{1,mn0}^{(1)} + b_{mn} f_{1,mn0}^{(2)}\right] dk, \quad (12)$$

where the functions $f_{mn0}^{(q)}$, $q = 1,2$, are defined by the formulas for FWD case, for $i = 0$, for the water regions $\mathcal{F}_0$ and $\mathcal{F}_1$.

The expressions (13), (7) and (11), (12) are for the total free surface elevation for FWD and IWD cases. We can subtract the incident field and use the second terms in the right sides of these exspressions, representing ζ^{pm}, to study the initiated wave pattern, generated by the plate motion.

The free surface elevation (FWD case)

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - \pi i \sum_{m=1}^{M} \left(\mathcal{D}\kappa_m^4 - \mu\right)$$
$$\times \sum_{i=0}^{M-3} \frac{k_i^2 K}{(k_i^2 h - K^2 h + K)(k_i^2 - \kappa_m^2)} \left[a_{mn} f_{mni}^{(1)} + b_{mn} f_{mni}^{(2)}\right] dk, \quad (13)$$

where introduced functions $f_{mni}^{(q)}$ have the following expressions in the open water $\mathcal{F}_0$ and in the gap $\mathcal{F}_1$

$$f_{mni}^{(q)} = \begin{cases} r_0 H_n^{(1)}(k_i\rho) \left[k J_{n+1}(k_i r_0) H_n^{(q)}(\kappa_m r_0) - \kappa_m J_n(k_i r_0) H_{n+1}^{(q)}(\kappa_m r_0)\right] & \text{in } \mathcal{F}_0, \\ r_1 J_n(k_i\rho) \left[k H_{n+1}^{(1)}(k_i r_1) H_n^{(q)}(\kappa_m r_1) - \kappa_m H_n^{(1)}(k_i r_1) H_{n+1}^{(q)}(\kappa_m r_1)\right] & \text{in } \mathcal{F}_1, \end{cases} \quad (14)$$

where $q = 1,2$ and $i = 0..M-3$.