

HYDROELASTIC ANALYSIS OF FLOATING PLATE OF FINITE DRAFT

Alexey I. Andrianov¹ & Aad J. Hermans²

Delft University of Technology
Delft Institute of Applied Mathematics
Mekelweg 4, 2628 CD Delft, The Netherlands

Summary

The paper considers the diffraction of incident surface water waves by a very large floating platform. The problem is solved for the case of finite water depth. The platform is of finite draft and modeled by an elastic thin plate. We consider the half-plane problem; the approach may be extended to the plates of other horizontal planform. The deflection of the plate is represented as the series of the solutions with respect to draft; we present the first and second term. Each of these terms can be written as the series of exponential functions. We obtain reduced wavenumbers and derive the sets of equations for corresponded wave amplitudes. Results obtained for the plate of finite draft are compared to the results obtained with use of zero-thickness assumption.

Keywords: plate-water interaction, diffraction, incident waves, hydroelastic response, deflection, dispersion relation, integro-differential equation, Green's function, very large floating platform, finite draft, three-dimensional analysis.

Abbreviations: VLFP - very large floating platform, IDE - integro-differential equation.

1 Introduction

The plate-water interaction is an important subject of the hydrodynamics, widely studied during last years. A very detailed literature survey for the hydroelastic analysis of VLFP has been published recently by Watanabe et al. [1]. There are several approaches used to describe the interaction between VLFP and surface water waves. Usually, VLFP is modeled by a thin elastic plate.

The solution for the case, when the plate thickness is assumed to be zero, has been derived by authors, published in [2] and presented at 17th IWWF. In this paper we study the plate of finite thickness and draft.

2 Formulation

The semi-infinite plate of finite draft covers the part of the surface of the water, which is assumed to be an ideal incompressible fluid of finite and constant depth. The plate deflection is generated by incoming surface waves propagated in positive x -direction. The geometry and the coordinate system chosen are shown in figure 1.

Parameters of the plate and characteristics of the material: ρ_p - density, m - mass of unit area, D - flexural rigidity, h_p - thickness, d - draft. The mass per unit area is $m = \rho_p h_p$, D is the flexural rigidity, expressed in terms of the Young's modulus E , Poisson's ratio ν and the plate thickness h_p , $D = Eh_p^3/12(1 - \nu^2)$.

The water and wave parameters: ρ_w - density, h - depth, λ - wavelength, k_0 - wavenumber, ω - wave frequency, A - wave height. We take the zero angle of incidence β ; transition from the perpendicular waves case to the oblique waves case is not difficult.

Further, $\mathcal{D} = D/\rho_w g$, $\mu = m\omega^2/\rho_w g$ are introduced structural parameters, constant as isotropic plate is considered; the flexural rigidity and bending stiffness of the plate are constant.

The wave height A is smaller than the plate thickness h_p . Therefore, there is no cavity between wetted surface of the plate and water surface.

With the usual assumptions of an ideal fluid and small amplitudes, the velocity potential can be written in the form $\Phi(x, y, z, t) = \phi(x, y, z)e^{-i\omega t}$. The potential $\phi(x, y, z)$ is a solution of the Laplace equation in the fluid, $-h < z < 0$,

$$\Delta\phi = 0, \quad (1)$$

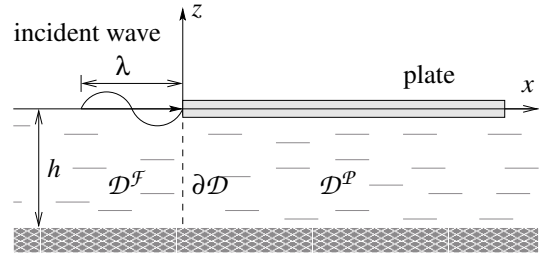


Figure 1: definition sketch of the problem.

together with the boundary conditions on the free surface $z = 0$, on the bottom $z = -h$, $\phi_z = 0$, and on the floating plate wetted surfaces, which are horizontal \mathcal{P} ($x > 0, z = -d$) and vertical VS ($x = 0, -d < z < 0$), see, e.g., [1].

Using the thin plate theory, we obtain the following expression for the total potential ϕ at the plate area

$$(\mathcal{D}\Delta^2 - \mu + 1)\phi_z - K\phi = 0, \quad (2)$$

where $K = \omega^2/g$. For the open water area, we have

$$\phi_z - K\phi = 0. \quad (3)$$

The potential of the undisturbed incident wave ϕ^{inc} is given by

$$\phi^{\text{inc}}(x, z) = \frac{\cosh k_0(z+h) gA}{\cosh k_0 h} \frac{e^{ik_0 x}}{i\omega}. \quad (4)$$

For the water of finite depth, the wavenumber k_0 is the only real solution of the water dispersion relation

$$k_i \tanh k_i h = K. \quad (5)$$

The wavelength of incoming waves is $\lambda = 2\pi/k_0$. The diffraction potential, which equals $\phi - \phi^{\text{inc}}$, must satisfy the Sommerfeld radiation condition at infinity.

The linearized free surface kinematic condition gives us the following relation between the potential and deflection

$$\phi_z(x, z) = -i\omega w(x). \quad (6)$$

The free horizontal edge of the plate is free of vertical forces, bending and twisting moments, and the free edge conditions at $y = 0$ are

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^3 w}{\partial x^3} = 0, \quad (7)$$

as the direction of the normal n coincides with the direction of x -axis. The condition at the vertical surface of the plate edge $x = 0$ has the form

$$\phi_x = 0. \quad (8)$$

¹A.I.Andrianov@EWI.TUdelft.Nl, www.andrianov.org

²A.J.Hermans@EWI.TUdelft.Nl, ta.twi.tudelft.nl/mf/users/hermans

3 Solution

With use of the thin plate theory and Green's theorem we obtain integral equation for the potential and, further, IDE for the plate deflection. The key idea of our approach is adding of a correction, which is dependent on draft, to our solution [2], obtained with the zero-thickness assumption.

As in the case of zero draft [2], we write the deflection of the plate of finite draft in the form

$$w(x) = \sum_{n=0}^{\infty} a_n e^{i\kappa_n x}, \quad (9)$$

where the amplitudes a_n and reduced wavenumber κ_n are unknown. Each term of the series represents corresponded wave mode. Due to the convergence of the series in (9) a finite number, M , of wave modes is taken into account. However, now the amplitude is written as a power series with respect to small value of the draft as follows

$$a_n = a_n^{(0)} + da_n^{(1)} + O(d^2). \quad (10)$$

To avoid secular behavior of the amplitudes, as in the PLK method [4], we also expand the reduced wavenumber κ_n as power series

$$\kappa_n = \kappa_n^{(0)} \left(1 + d\kappa_n^{(1)}\right) + O(d^2). \quad (11)$$

Terms $a_n^{(0)}$, $a_n^{(1)}$ and $\kappa_n^{(0)}$, $\kappa_n^{(1)}$ of the corresponded series are of zero and first draft orders respectively. The plate deflection (9) may be rewritten as the sum of zero order draft solution (with a correction term in the exponential function) and first order draft solution

$$w(x) = w^{(0)}(x) + dw^{(1)}(x), \quad (12)$$

where $w^{(q)}(x)$, $q = 0, 1$, has the form

$$w^{(q)}(x) = \sum_{n=0}^M a_n^{(q)} e^{i\kappa_n^{(0)}(1+d\kappa_n^{(1)})x}. \quad (13)$$

For the case when thickness is assumed to be zero [2], the deflection $w^{(0*)}(x)$ is represented in the form

$$w^{(0*)}(x) = \sum_{n=0}^M a_n^{(0)} e^{i\kappa_n^{(0)}x}. \quad (14)$$

We introduce the Green's function for a source within the fluid, and apply the Green's theorem for the potential in open water and plate areas. The total potential ϕ satisfies

$$2\pi\phi = 2\pi\phi^{\text{inc}} + \int_{VS} \left(\phi \frac{\partial \mathcal{G}}{\partial n} - \mathcal{G} \frac{\partial \phi}{\partial n} \right) d\zeta + \int_P \left(\phi \frac{\partial \mathcal{G}}{\partial n} - \mathcal{G} \frac{\partial \phi}{\partial n} \right) d\xi. \quad (15)$$

The normal derivative of the potential, ϕ_n , for vertical surface, as the normal coincides with x -axis, becomes ϕ_x , that equals 0 due to (8). For horizontal surface, whereas the normal coincides with z -axis, the potential normal derivative becomes ϕ_z . Thus, for the potential in the plate area the following approximate integral equation is derived

$$\begin{aligned} 2\pi\phi(x, -d) &= 2\pi\phi^{\text{inc}} - d\phi(0, 0) \frac{\partial \mathcal{G}(x, 0; 0, 0)}{\partial \xi} \\ &+ \int_0^\infty \left(\phi(\xi, -d) \frac{\partial \mathcal{G}(x, 0; \xi, 0)}{\partial \zeta} - \mathcal{G}(x, 0; \xi, 0) \frac{\partial \phi(\xi, -d)}{\partial \zeta} \right) d\xi \\ &+ 2d \int_0^\infty \mathcal{G}(x, 0; \xi, 0) \left(\frac{\partial^2 \phi(\xi, -d)}{\partial \xi^2} + K \frac{\partial \phi(\xi, -d)}{\partial \zeta} \right) d\xi, \end{aligned} \quad (16)$$

where the two-dimensional Green's function at the free surface has the form, see [3],

$$\mathcal{G}(x, 0; \xi, 0) = - \int_0^\infty \frac{\cosh kh}{k \sinh kh - K \cosh kh} e^{ik(x-\xi)} dk. \quad (17)$$

Using the relations between the potential $\phi(x, -d)$ and its derivatives and the deflection $w(x)$, we obtain the following integro-differential equation for the plate deflection

$$\begin{aligned} &\left\{ \mathcal{D} \frac{\partial^4}{\partial x^4} - \mu + 1 \right\} w(x) \\ &- \frac{K}{2\pi} \int_0^\infty \mathcal{G}(x, 0; \xi, 0) \left\{ \mathcal{D} \frac{\partial^4}{\partial \xi^4} - \mu \right\} w(\xi) d\xi = F_0, \end{aligned} \quad (18)$$

where the function F_0 contains terms from the first and third lines in the right-hand side of (16). If we insert expressions for the deflection (9) and Green's function (17) into IDE (18), keeping a_n and κ_n in the general forms, the following equation is derived

$$\begin{aligned} &\sum_{n=0}^M (\mathcal{D}\kappa_n^4 - \mu + 1) a_n e^{i\kappa_n x} + \frac{K}{2\pi} \int_0^\infty \sum_{n=0}^M (\mathcal{D}\kappa_n^4 - \mu) a_n e^{i\kappa_n \xi} \\ &\times \int_0^\infty \frac{\cosh kh}{k \sinh kh - K \cosh kh} e^{ik(x-\xi)} dk d\xi = F_0. \end{aligned} \quad (19)$$

Next, the integration with respect to ξ has to be done. The k -integral can be solved by means of the residues at the poles $k = \kappa_n$ and $k = k_i$ of the complex plane

$$\begin{aligned} &\sum_{n=0}^M (\mathcal{D}\kappa_n^4 - \mu + 1) a_n e^{i\kappa_n x} + K \sum_{n=0}^M (\mathcal{D}\kappa_n^4 - \mu) a_n \\ &\times \left(\frac{e^{i\kappa_n x}}{\kappa_n \tanh \kappa_n h - K} + \sum_{i=0}^{M-2} \frac{k_i e^{ik_i x}}{(\kappa_n - k_i) \mathcal{K}_i} \right) = F_0, \end{aligned} \quad (20)$$

where the introduced functions \mathcal{K}_i for $i = 0, \dots, M-2$ are

$$\mathcal{K}_i = K(1 - Kh) + k_i^2 h. \quad (21)$$

Then we rewrite the amplitudes a_n in the form (10) and obtain the following extended expression

$$\begin{aligned} &\sum_{n=0}^M (\mathcal{D}\kappa_n^4 - \mu + 1) \left(a_n^{(0)} + da_n^{(1)} \right) e^{i\kappa_n x} \\ &+ K \sum_{n=0}^M (\mathcal{D}\kappa_n^4 - \mu) \left(a_n^{(0)} + da_n^{(1)} \right) \frac{e^{i\kappa_n x}}{\kappa_n \tanh \kappa_n h - K} = F, \end{aligned} \quad (22)$$

where the function F is the sum of the original function F_0 plus the contribution of the poles $k = k_i$ in (22):

$$\begin{aligned} F &= A e^{ik_0 x} - K \sum_{n=0}^M (\mathcal{D}\kappa_n^4 - \mu) \left(a_n^{(0)} + da_n^{(1)} \right) \sum_{i=0}^{M-2} \frac{k_i e^{ik_i x}}{(\kappa_n - k_i) \mathcal{K}_i} \\ &+ d \sum_{n=0}^M (\mathcal{D}\kappa_n^4 - \mu + 1) \left(a_n^{(0)} + da_n^{(1)} \right) \sum_{i=0}^{M-2} \frac{k_i^2 e^{ik_i x}}{\mathcal{K}_i} \\ &+ 2d \sum_{n=0}^M [(\mathcal{D}\kappa_n^4 - \mu + 1) \kappa_n^2 - K^2] \left(a_n^{(0)} + da_n^{(1)} \right) \\ &\times \left(\sum_{i=0}^{M-2} \frac{k_i e^{ik_i x}}{(\kappa_n - k_i) \mathcal{K}_i} + \frac{e^{i\kappa_n x}}{\kappa_n \tanh \kappa_n h - K} \right). \end{aligned} \quad (23)$$

4 Zero draft order

Now we distinguish terms of zero draft order, that allows us to find $\kappa_n^{(0)}$ and, later, $a_n^{(0)}$. Analyzing the left-hand side in (22) and collecting $O(d^0)$ terms, we obtain

$$Ae^{ik_0x} - K \sum_{n=0}^M (\mathcal{D}\kappa_n^4 - \mu) a_n^{(0)} \sum_{i=0}^{M-2} \frac{k_i e^{ik_i x}}{(\kappa_n - k_i) \mathcal{K}_i}, \quad (24)$$

where the function $\mathcal{F}(\kappa_n)$ has the form

$$\mathcal{F}(\kappa_n) = \frac{(\mathcal{D}\kappa_n^4 - \mu + 1) \kappa_n \tanh \kappa_n h - K}{\kappa_n \tanh \kappa_n h - K}. \quad (25)$$

In the nominator of the function $\mathcal{F}(\kappa_n)$ we have the dispersion relation in the plate region of zero draft order, which is

$$(\mathcal{D}\kappa_n^{(0)4} - \mu + 1) \kappa_n^{(0)} \tanh \kappa_n^{(0)} h = K. \quad (26)$$

Making M the truncation parameter of the problem, we take into account $M + 1$ roots $\kappa_n^{(0)}$ of the plate dispersion relation (26) and $M - 1$ roots k_i of the water dispersion relation (5).

Then, the set of equations for the amplitudes $a_n^{(0)}$ may be obtained if we consider the coefficients of the exponential function $e^{ik_i x}$. The obtained $M - 1$ equations are

$$\sum_{n=0}^M (\mathcal{D}\kappa_n^{(0)4} - \mu) \frac{k_i K a_n^{(0)}}{\mathcal{K}_i (\kappa_n^{(0)} - k_i)} = A_i, \quad (27)$$

where $i = 0, \dots, M - 2$, and $A_0 = -A$, $A_i = 0$ for $i > 0$. The rest of equations are obtained from the free edge conditions (7):

$$\sum_{n=0}^M \kappa_n^{(0)2} a_n^{(0)} = 0, \quad \sum_{n=0}^M \kappa_n^{(0)3} a_n^{(0)} = 0. \quad (28)$$

The set for $a_n^{(0)}$ consists of $M + 1$ equations (27)-(28), having exactly the same form with one derived for the case of zero-thickness assumption in [2]. In such a way, the deflection function $w^{(0*)}(x)$ can be computed by formula (14).

5 First draft order

Next, we consider the terms of first draft order in the extended IDE (22). Terms, containing $\kappa_n - k_i$ in the denominator in (23), result in extra terms of $O(d^1)$. The first draft order equation becomes

$$\begin{aligned} & \sum_{n=0}^M \left(\mathcal{F}(\kappa_n^{(0)}) a_n^{(1)} + \frac{\partial \mathcal{F}(\kappa_n^{(0)})}{\partial \kappa_n} \kappa_n^{(0)} \kappa_n^{(1)} a_n^{(0)} \right) \frac{e^{i\kappa_n x}}{\kappa_n \tanh \kappa_n h - K} = \\ & -K \sum_{n=0}^M (\mathcal{D}\kappa_n^{(0)4} - \mu) \sum_{i=0}^{M-2} \frac{e^{ik_i x}}{(\kappa_n^{(0)} - k_i) \mathcal{K}_i} \left(a_n^{(1)} - \frac{\kappa_n^{(0)} \kappa_n^{(1)} a_n^{(0)}}{(\kappa_n^{(0)} - k_i)} \right) \\ & + \sum_{n=0}^M (\mathcal{D}\kappa_n^{(0)4} - \mu + 1) \sum_{i=0}^{M-2} \frac{k_i^2 e^{ik_i x}}{\mathcal{K}_i} a_n^{(0)} \\ & + 2 \sum_{n=0}^M \left[(\mathcal{D}\kappa_n^{(0)4} - \mu + 1) \kappa_n^{(0)2} - K^2 \right] \sum_{i=0}^{M-2} \frac{k_i e^{ik_i x} a_n^{(0)}}{(\kappa_n^{(0)} - k_i) \mathcal{K}_i} \\ & + 2 \sum_{n=0}^M \left[(\mathcal{D}\kappa_n^{(0)4} - \mu + 1) \kappa_n^{(0)2} - K^2 \right] \frac{e^{i\kappa_n x} a_n^{(0)}}{\kappa_n \tanh \kappa_n h - K}. \quad (29) \end{aligned}$$

Function $\mathcal{F}(\kappa_n)$, having the form (25), has been rewritten as

$$\mathcal{F}(\kappa_n) = \mathcal{F}(\kappa_n^{(0)}) + d \frac{\partial \mathcal{F}(\kappa_n^{(0)})}{\partial \kappa_n} \kappa_n^{(0)} \kappa_n^{(1)} + O(d^2). \quad (30)$$

Doing some operations and considering the coefficients of $e^{i\kappa_n x}$, the following relation is obtained for reduced wavenumbers of first draft order $\kappa_n^{(1)}$

$$\kappa_n^{(1)} = \frac{2 \left[(\mathcal{D}\kappa_n^{(0)4} - \mu + 1) \kappa_n^{(0)2} - K^2 \right]}{\kappa_n^{(0)} Q(\kappa_n^{(0)})}, \quad (31)$$

$$\begin{aligned} Q(\kappa_n^{(0)}) &= (\mathcal{D}\kappa_n^{(0)4} - \mu + 1) \kappa_n^{(0)} h \\ &+ (5\mathcal{D}\kappa_n^{(0)4} - \mu + 1) \tanh \kappa_n^{(0)} h - Kh \tanh \kappa_n^{(0)} h. \quad (32) \end{aligned}$$

It is seen from (31), that $\kappa_n^{(1)} \sim O(2/h)$, and also that for realistic values of the draft and wavelength $d\kappa_n^{(1)} < 1$, that one can expect. The nature of $\kappa_n^{(1)}$ is the same with corresponded $\kappa_n^{(0)}$ for $n = 0, 1, 2$: they are purely real when $n = 0$, and are complex roots when $n = 1, 2$. But for $n > 2$ $\kappa_n^{(1)}$ are purely real, while $\kappa_n^{(0)}$ are purely imaginary roots.

Knowing $\kappa_n^{(1)}$ and $a_n^{(0)}$, the deflection function $w^{(0)}$ is computed by formula (13). Then, the amplitudes $a_n^{(1)}$ may be found with use of (29), $a_n^{(0)}$, $\kappa_n^{(0)}$ and $\kappa_n^{(1)}$. The equations to determine $a_n^{(1)}$ have the following form for $i = 0, \dots, M - 2$

$$\begin{aligned} & K \sum_{n=0}^M (\mathcal{D}\kappa_n^{(0)4} - \mu) \frac{k_i e^{ik_i x}}{(\kappa_n^{(0)} - k_i) \mathcal{K}_i} a_n^{(1)} \\ &= K \sum_{n=0}^M (\mathcal{D}\kappa_n^{(0)4} - \mu) \frac{\kappa_n^{(0)} \kappa_n^{(1)} k_i e^{ik_i x}}{(\kappa_n^{(0)} - k_i)^2 \mathcal{K}_i} a_n^{(0)} \\ &+ \sum_{n=0}^M (\mathcal{D}\kappa_n^{(0)4} - \mu + 1) \frac{k_i^2 e^{ik_i x}}{\mathcal{K}_i} a_n^{(0)} \\ &+ 2 \sum_{n=0}^M \left[(\mathcal{D}\kappa_n^{(0)4} - \mu + 1) \kappa_n^{(0)2} - K^2 \right] \frac{k_i e^{ik_i x} a_n^{(0)}}{(\kappa_n^{(0)} - k_i) \mathcal{K}_i}, \quad (33) \end{aligned}$$

and are supplemented by the free edge conditions

$$\sum_{n=0}^M \kappa_n^{(0)2} a_n^{(1)} = -2 \sum_{n=0}^M \kappa_n^{(0)2} \kappa_n^{(1)} a_n^{(0)}, \quad (34)$$

$$\sum_{n=0}^M \kappa_n^{(0)3} a_n^{(1)} = -3 \sum_{n=0}^M \kappa_n^{(0)3} \kappa_n^{(1)} a_n^{(0)}. \quad (35)$$

We may note, that coefficients of the amplitudes $a_n^{(0)}$ and $a_n^{(1)}$ in the corresponded sets of $M + 1$ equations are the same. The set of equations (33)-(35) of first draft order allows us to find the amplitudes $a_n^{(1)}$ and, then, the deflection term $w^{(1)}$. Hence, the total deflection can be computed by formula (9).

6 Results & discussion

The number of the roots $\kappa_n^{(0)}$ and, correspondingly, $\kappa_n^{(1)}$, which are taken into account, is $M = 30$. It provides sufficient accuracy. We take Poisson's ratio $\nu = 0.25$, ratio $m/\rho_w = 0.25$ m, and wave height $A = 1$ m as constant. Hence, results shown

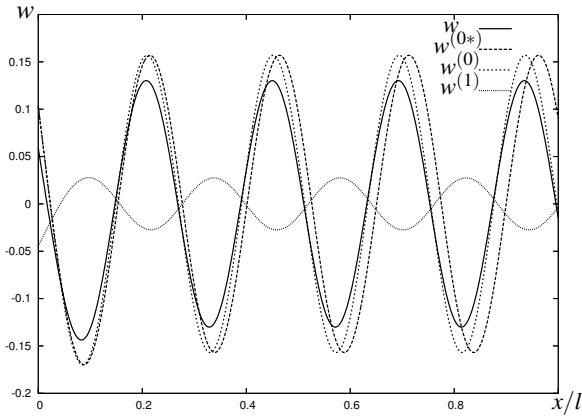


Figure 2: w , $w^{(0*)}$, $w^{(0)}$, $w^{(1)}$; for $h = 10$ m, $\mathcal{D} = 10^7$ m⁴, $l = 1000$ m, $\lambda = 200$ m, $d = 1$ m.

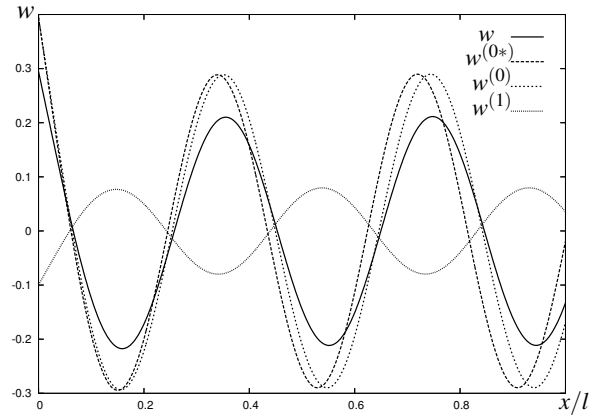


Figure 4: w , $w^{(0*)}$, $w^{(0)}$, $w^{(1)}$; for $h = 100$ m, $\mathcal{D} = 10^5$ m⁴, $l = 300$ m, $\lambda = 60$ m, $d = 1$ m.

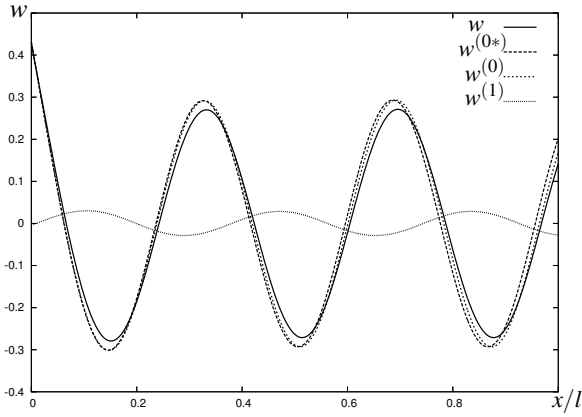


Figure 3: w , $w^{(0*)}$, $w^{(0)}$, $w^{(1)}$; as in figure 2, for $h = 100$ m.

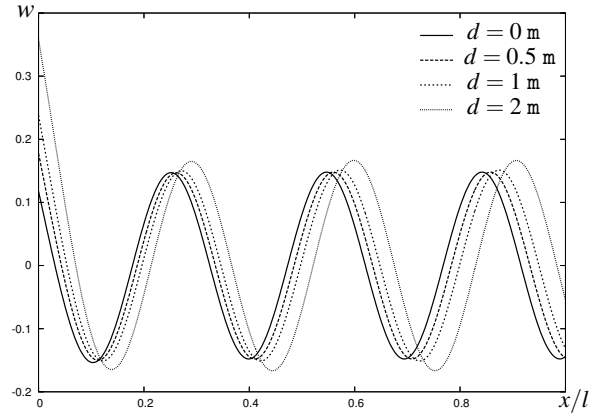


Figure 5: plate deflection w ; for different draft values and $h = 100$ m, $\mathcal{D} = 10^7$ m⁴, $l = 1000$ m, $\lambda = 100$ m.

are for the real part of the deflection, normalized by the wave height. Results for w are plotted against x/l , where l is the length of the plate part considered. Four curves in figures 2-4 represent: the total deflection w , deflection for the case of zero draft (zero-thickness assumption) $w^{(0*)}$, the deflection terms of zero $w^{(0)}$ and first $w^{(1)}$ draft orders.

In figures 2-3 results are shown for the realistic values of the plate rigidity (as for VLFP) for shallow and rather deep water. In figure 4 results are plotted for low rigidity (an ice field). We may see the shift, when $w^{(0*)}$ and $w^{(0)}$ are compared; the direction of shift may be different for shallow and deep water (to the left and to the right respectively).

Results for the total deflection w for different values of the draft are given in figure 5. The difference between the total deflection w for finite draft and the deflection function $w^{(0*)}$, and direction and value of the shift ($w^{(0)}$ and $w^{(0*)}$) is highly dependent on the draft value. The draft has sufficient influence on the results, especially for the plate of low rigidity. For realistic values of the plate rigidity and draft (as for VLFP), water depth and wavelength, the influence is not so large though.

7 Conclusions & summary

The problem of the diffraction of surface water waves on the floating flexible plate of finite draft is solved. The analytical and numerical study for the plate hydroelastic behavior is presented. The plate deflection is represented as the series of the solutions with respect to the draft. The problem is solved with use of the thin plate theory, Green's theorem and derived IDE. The correction, appeared due to nonzero thickness, is studied. Other unknown parameters, such as the free surface elevation, reflection and transmission coefficients may be studied as well.

For the case of shallow water this problem may be solved with use of the Stoker approximation theory.

This is the new extension of our method, which has previously been applied to the plates of (assumed) zero thickness. The approach presented may be rather straightforward extended to the case of oblique incident waves or to the strip of finite width and infinite length. Also, the approach can be extended to other horizontal planforms of the plate: circular and ring-shaped, quarter-infinite, rectangular finite, etc.

More details, information and results will be presented at the Workshop.

References

- [1] Watanabe E., Utsunomiya T. & Wang C.M., Hydroelastic Analysis of Pontoon-Type VLFS: a Literature Survey. *Engineering Structures*, 2004; 26 (2): p.245-56.
- [2] Andrianov A.I. & Hermans A.J., The Influence of Water Depth on the Hydroelastic Response of a Very Large Floating Platform. *Marine Structures*, 2003; 16 (5): p.355-71.
- [3] Hermans A.J., The Ray Method for the Deflection of a Floating Flexible Platform in Short Waves. *Journal of Fluids and Structures*, 2003; 17 (4): p.593-602.
- [4] Kevorkian J. & Cole J.D., Multiple Scale and Singular Perturbation Methods. Springer-Verlag, New York, 1996, 633p.